**Combining Two Automata by Intersection - Exercise**

*Problem*:

Recall that the intersection operation may be performed on two automata which is defined as follows:

Let $L_1 = L(M_1)$ and $L_2 = L(M_2)$ where

$M_1 = (Q, \sum, \delta_1, q_0, F_1)$ and

$M_2 = (P, \sum, \delta_2, p_0, F_2)$ are DFAs.

We construct:

$\hat{M} = (\hat{Q}, \sum, \hat{\delta}, (q_0, p_0), \hat{F})$

where

$\hat{Q} = Q \times P$

$\hat{\delta} : \hat{\delta}((q_i, p_j), a) = (q_k, p_l)$ where

$\delta(q_i, a) = q_k$ and

$\delta(p_j, a) = p_l$

$\hat{F} = \{(q_i, p_j) : q_i \in F_1, p_j \in F_2$

Then $\hat{M}$ accepts all $w$, $w \in L_1 \cap L_2$.

Since $\hat{M}$ is defined by a DFA, $L_1 \cap L_2$ is regular.

Find $L_1 \cap L_2$ using JFLAP where

$L_1$ = all words with an aa

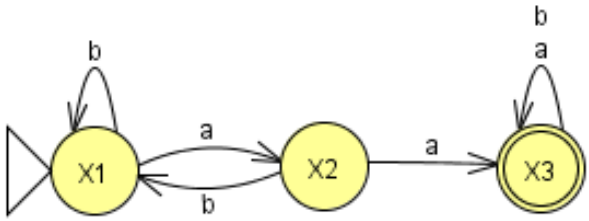
$L_2$ = all words with an even number of a's

The alphabet is over {a, b}.

*Solution*:

We consider $L_1$ first where all words must end with an aa. A DFA for this language would be:

$M_1$ =



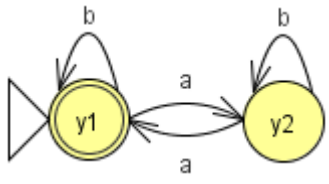

We will refer to this DFA as $M_1$. Create the DFA in JFLAP.

The corresponding regular expressions are:

$r_1 = (a+b)^*aa(a+b)^*$ or $(b^*aab^*)^*$

Next, we consider $L_2$ where all words contain an even number of a's. A DFA for this language would be:

$M_2$ =



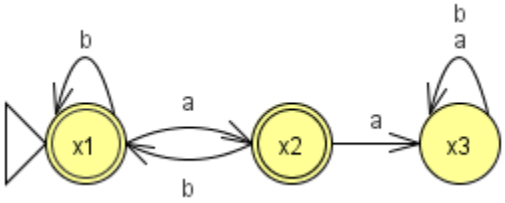

The corresponding regular expressions are:

$r_2 = (b^*ab^*ab^*)^*$ or $(b+ab^*a)^*$

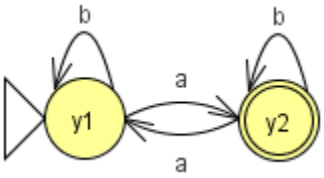
In this exercise, we will use **De Morgan's law** to perform the intersection. Recall that

$$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$$

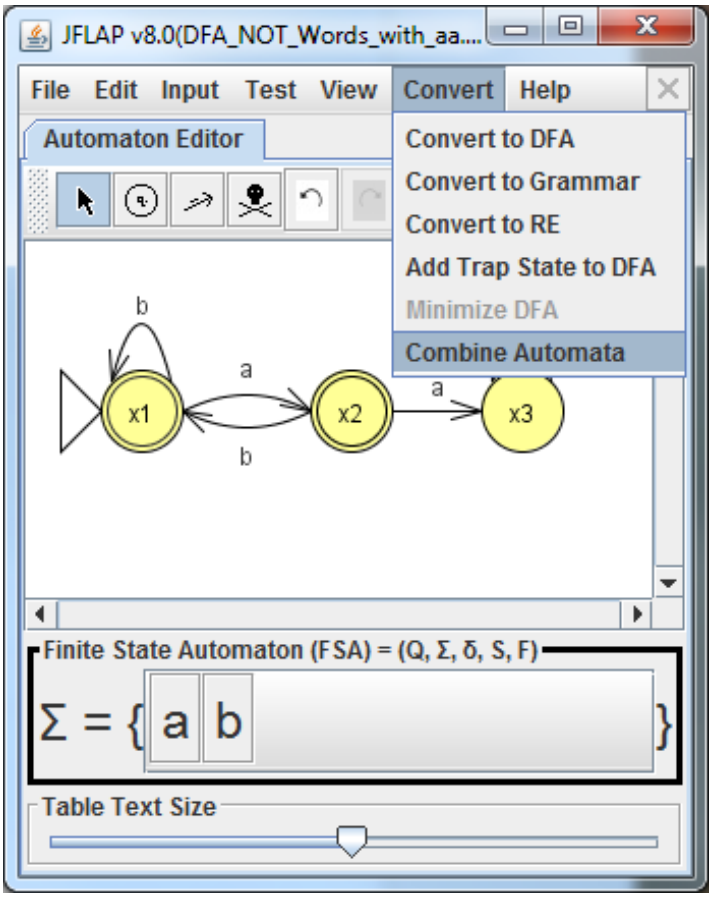
So we will start by $\overline{L_1}$. The complement of $L_1$ is:



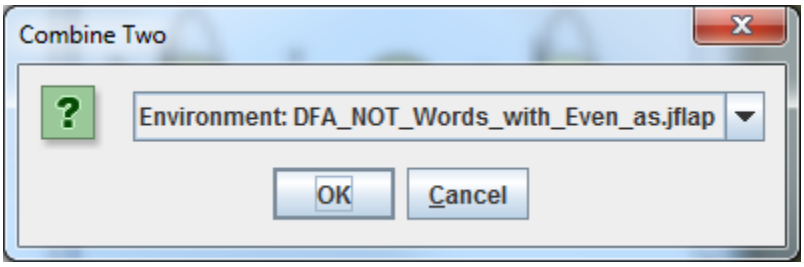

And the complement of $L_2$, $\overline{L_2}$ is:



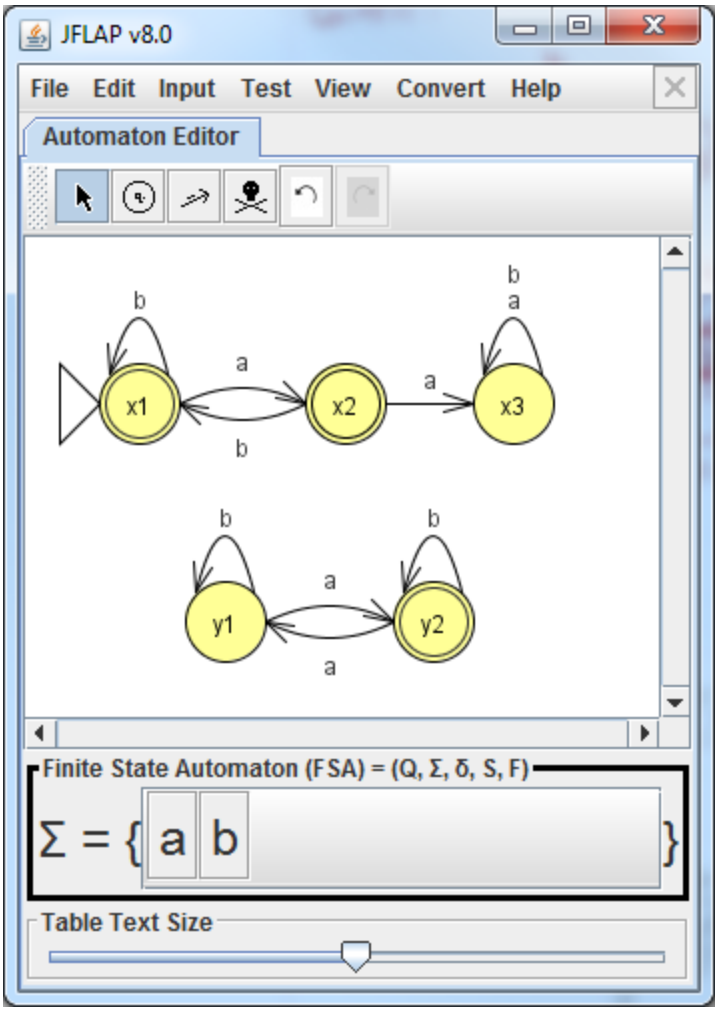

Now, we will combine the two separate DFAs in JFLAP to start forming the union. Do this by starting with one of the two DFAs (doesn't matter which) and clicking on *Convert* > *Combine Automata*.




A dialog box asking for the name of the second automata will show. Select the correct filename of the second automata if it isn't selected yet.

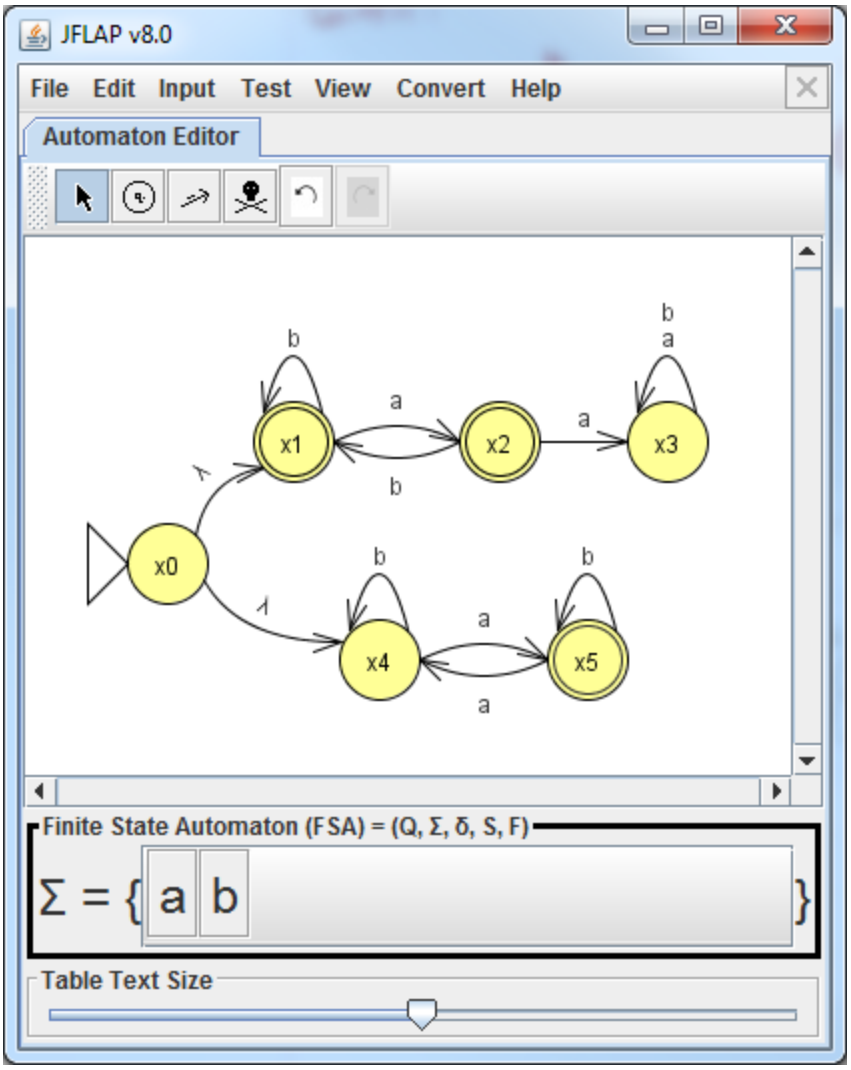



Click OK. A third JFLAP window should pop up showing the two DFAs in the same JFLAP window. Adjust the placement of the states so that it is readable.

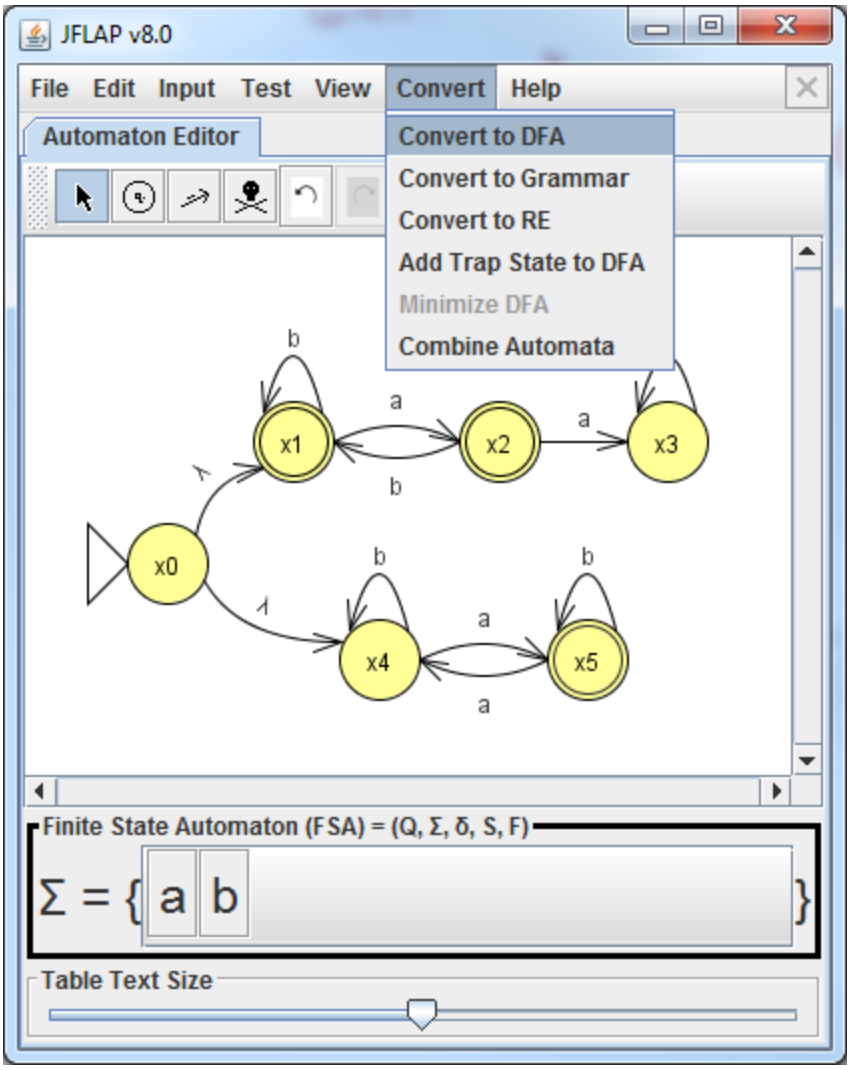



We are now ready to connect the two DFAs to form the union, $\overline{L_1} \cup \overline{L_2}$, (applying De Morgan's law). Do this by:

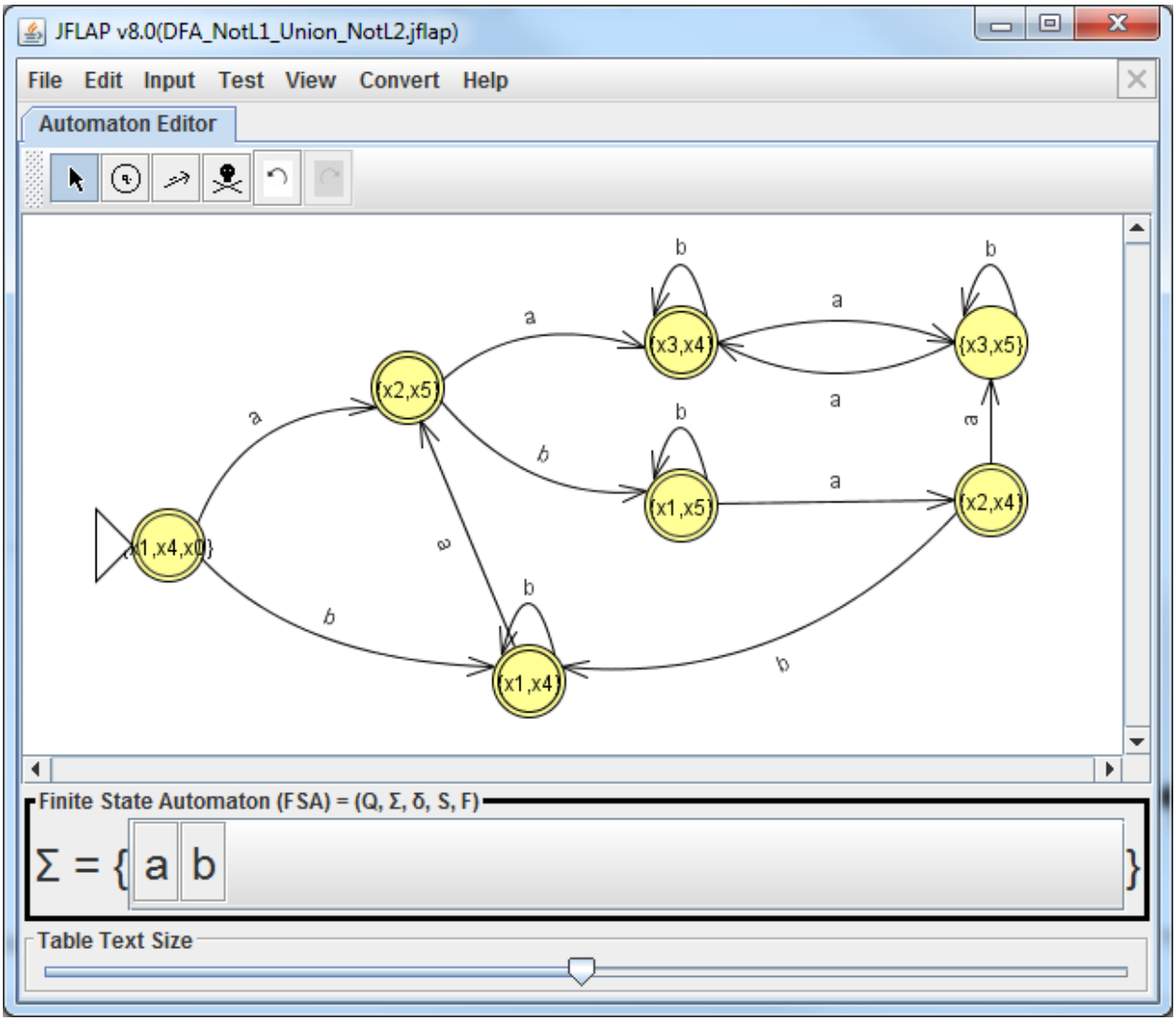
1. renaming y1 to x4 and y2 to x5,
2. making the start state a non-start state,
3. adding a new start state named x0,
4. connecting x0 to x1 and x4 with a lambda transition



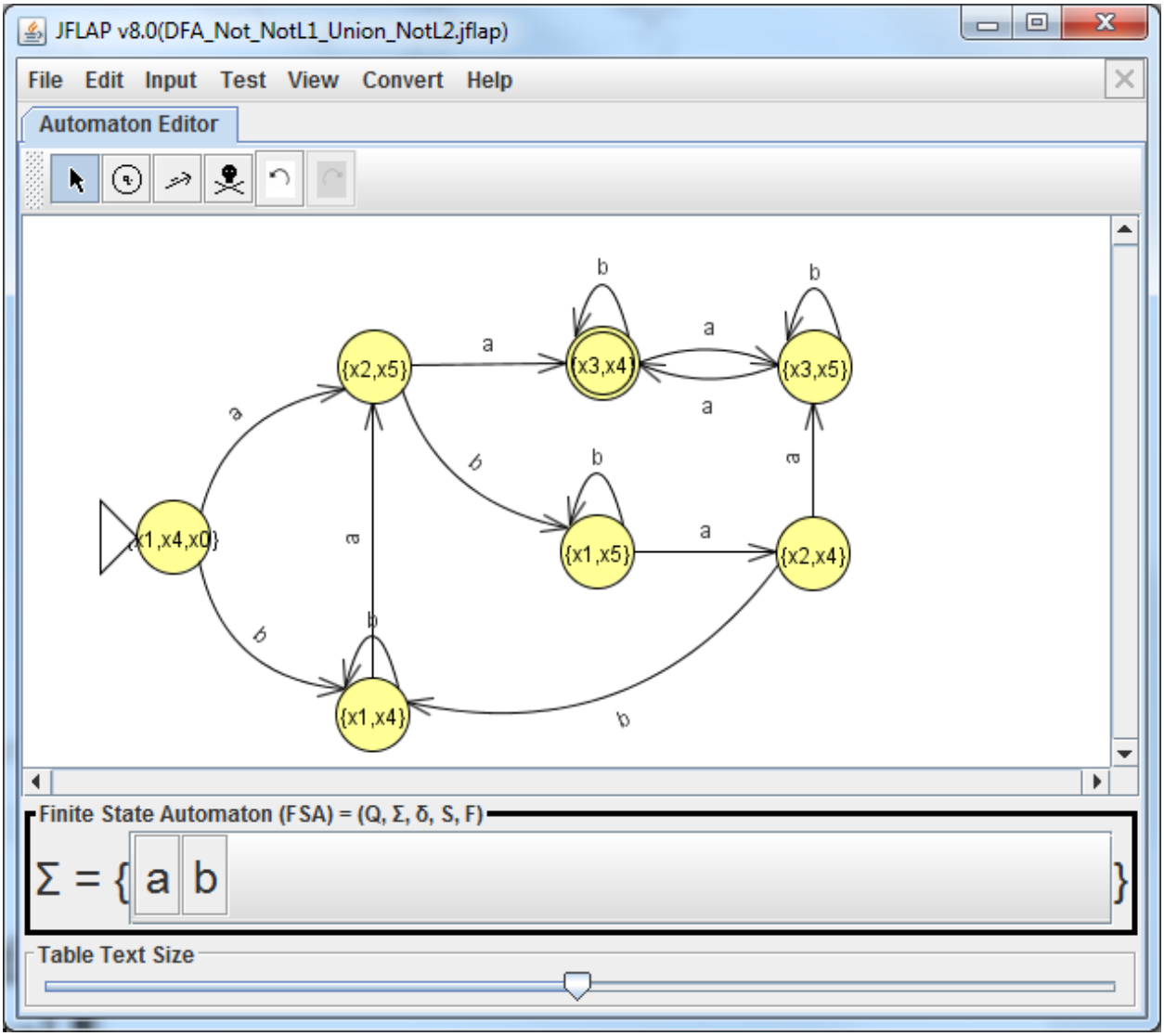

We now have an NFA for$\overline{L_1} \cup \overline{L_2}$. In order to find its complement, $\overline{\overline{L_1} \cup \overline{L_2}}$, we need to convert this to a DFA. Click Convert > Convert to DFA.




When a new pane shows on the right, choose *Complete*, then *Export*. This places the resulting DFA in a new window. Again, re-position the states so that the DFA is readable.




The last step is to find its complement, $\overline{\overline{L_1} \cup \overline{L_2}}$. Take the complement of the resulting DFA by reversing the final states to non-final states.




We can also visually inspect that the final DFA accepts words that end with aa and must contain an even number of a.